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# Astrophysical Consequences of a Violation of the Strong Equivalence Principle

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A previous analysis of the consequences of a violation of the strong equivalence principle (SEP) for massive particles has now been extended to the case of photons and leads to the prediction that the photon number is conserved. Comparison of the predictions of our theory with observations, especially those on big-bang nucleosynthesis, shows that a violation of the SEP is not ruled out by the data available, and that further direct measurements are required.

## 1. INTRODUCTION

The strong equivalence principle (SEP) states that the outcome of any gravitational or nongravitational experiment is independent of where and when in the universe it is performed (Canuto and Goldman, 1982*a*), or, in particular, that in spite of changes in the global distribution of matter in the universe, no influence is to be felt by local gravitational and nongravitational experiments performed at different epochs. It can be shown (Canuto and Goldman, 1982*a*) that the SEP requires that all clocks in nature be equivalent, or that the ratio of their intrinsic units of time be constant. An SEP violation occurs when the ratio of the period of an atomic (or nuclear or weak) clock to that of a gravitational clock (such as a planet revolving about a star) is not constant in time. If by $\Delta t_E$ we denote a time interval recorded with a gravitational clock and by $\Delta t$ that recorded by an atomic clock, we characterize an SEP violation by a nonnull value of the time derivative $\dot{\beta}_a$ of the quantity $\beta_a$ defined by $\Delta t_E = \beta_a \Delta t$ (Canuto and Goldman, 1982*a*,*b*).

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The question whether or not the SEP is violated can be resolved directly by observations. For example, by using an atomic clock to record the flight time of photons to and from Mars, one can monitor the period of Mars orbiting the Sun, which is a gravitational clock. The analysis of the radar ranging data would then lead to a determination of $\dot{\beta}_a$ (Canuto *et al.*, in press).

To stimulate an observational search to test the SEP, it is necessary to show that an SEP violation is at least not ruled out by data already existing. To carry out the analysis, one needs a theoretical scheme and in particular to know whether an SEP violation would affect both gravitational and nongravitational physics, as is possible in principle. Before our work, it was generally assumed that an SEP violation would affect only gravitational phenomena and that all nongravitational physics is independent of it. To be specific, it was assumed that both one- and many-particle relations for fermions and bosons remain independent of $\beta_a$ even when $\dot{\beta}_a \neq 0$. The prototype of such theories is that of Brans and Dicke.

We have recently constructed an SEP-violating framework (Canuto and Goldman, 1982*a*) that does not require that an SEP violation should affect only gravitational phenomena.

Two main results have emerged. (1) Where massive particles are concerned, while one-particle relations remain unchanged, many-body relations (for example, the relation between pressure $P$ and density $\rho$) are affected by $\beta_a$, in contrast with the assumptions of previous theories, where this did not occur. (2) In order for atomic and gravitational clocks to run at different rates, the relation between the gravitational constant $G$ and the function $\beta_a$ must be of the form $G \sim \beta_a^{-g}$, with $g = 2$.

Our previous work (Canuto and Goldman, 1982*a*) did not deal with photons, but we have now been able to extend it so as to arrive at a framework for photons which is valid for any $g$. In so doing, we discover that our previous photon theory (Canuto and Hsieh, 1979) holds only for $g = 1$. That theory is therefore no longer tenable because it is inconsistent with the condition (2) above that for the SEP to be violated, $g$ must be 2. We stress that the $g = 1$ theory must be abandoned not for observational reasons, but on the grounds of theoretical consistency. (The implications of the two photon theories for the 3 K radiation are compared in what follows.)

The new photon theory allows us to construct five new tests of SEP violation. Together with previous tests of a cosmological nature (Canuto *et al.*, 1979; Canuto and Owen, 1979), we conclude that an SEP violation has been found to be compatible with a wide range of data from astrophysics to geophysics. The results do not prove that an SEP violation must occur, but they constitute the assurance required for an observational search to be undertaken (Canuto *et al.*, in press). [As in Canuto and Goldman (1982*a*), $\beta$ denotes general units; in atomic units, AU, $\beta = \beta_a$; in gravitational units, EU, $\beta = 1$.]

## 2. PHOTONS

### 2.1. Single-Particle Relations

We begin with the derivation of the equation for the propagation of photons using the equation of motion in terms of the four-velocity $u^\alpha$ of a test particle (Canuto and Goldman, 1982*a*) of mass $\mu$,

$$u^\alpha_{;\nu}u^\nu + \frac{(\mu\beta^{2-g})_{,\nu}}{(\mu\beta^{2-g})}\Delta^{\alpha\nu} = 0 \tag{1}$$

The second term in equation (1), involving the "projection operator" $\Delta_{\alpha\nu} = u_\alpha u_\nu - g_{\alpha\nu}$ ($\Delta_{\alpha\nu}u^\alpha u^\nu = 0$), is a deviation from the standard geodesic equation $u^\alpha_{;\nu}u^\nu = 0$ (where the semicolon denotes the covariant derivative) arising from the possible nonconstancy of $\mu$, $\beta$, and $G$.

Introducing the momentum $p_\alpha = \mu u_\alpha$ (with $c = 1$), where $p^\alpha p_\alpha = \mu^2$, we can rewrite equation (1) as

$$(\beta^{2-g}p_\alpha)_{;\nu}p^\nu - \tfrac{1}{2}\beta^{g-2}(\beta^{4-2g}p^\nu p_\nu)_{;\alpha} = 0 \tag{2}$$

For zero-rest-mass particles, $p^\nu p_\nu = 0$, this reduces to

$$(\beta^{2-g}p_\alpha)_{;\nu}p^\nu = 0 \tag{3}$$

which is equivalent to equation (3.10) of Canuto and Hsieh (1979) if one chooses $g = 1$.

To derive the energy-frequency ($\varepsilon_\gamma$, $\nu$) relation, note that the energy of a photon of momentum $p^\alpha$, as measured by an observer with velocity $u^\alpha$, is $\varepsilon_\gamma = u^\alpha p_\alpha$. As in the standard case, using a Robertson-Walker (RW) metric, equation (3) implies, for comoving observers $u^\alpha = \delta^\alpha_0$,

$$p_0 \sim \frac{\beta^{g-2}}{R} \qquad \text{and} \qquad \varepsilon_\gamma = \delta^\alpha_0\, p_\alpha = \frac{h\nu}{\beta^{2-g}} \tag{4}$$

where $h$ is a constant and where we have used the standard redshift relation $\nu R = \text{const}$, as required by the fact that the photon path is null. The energy-frequency relation (4) can be shown to hold true for a general metric. It can be seen that if $g = 1$, or $G \sim \beta^{-1}$, the relationship $\varepsilon_\gamma = h\nu/\beta$ derived in Canuto and Hsieh (1979) [equation (3.15)] is recovered.

Since we have shown (Canuto and Goldman (1982*a*) that $g$ must be 2, we conclude that the two relations in equation (4) are identical with those of standard theory.

## 2.2. Photons in a Beam

Let us now consider a beam of photons described by an energy-momentum tensor of the standard form $T_{\alpha\nu} = f u_\alpha u_\nu$, where the function $f$ will be determined later. In our framework, the possible nonconstancy of $\beta$ and $G$ changes the standard conservation law $T^{\alpha\nu}_{;\nu} = 0$ to [equation (4.1) of Canuto and Hsieh (1979)]

$$T^{\alpha\nu}_{;\nu} + (2-g)\frac{\beta_{,\nu}}{\beta} T^{\alpha\nu} - \frac{\beta^{\alpha}_{,}}{\beta} T^{\lambda}_{\lambda} = 0 \tag{5}$$

With the help of equation (3), we obtain

$$(fp^{\nu})_{;\nu} = 0 \tag{6}$$

To determine the function $f$, we note that the photon energy density $\rho_\gamma \equiv n_\gamma \varepsilon_\gamma$ (where $n_\gamma$ is the photon number density) measured by an observer with velocity $u^\alpha$ is given by $\rho_\gamma = T^{\alpha\nu} u_\alpha u_\nu = f(p^\alpha u_\alpha)^2 = f\varepsilon_\gamma^2$. Therefore, $f = n_\gamma/\varepsilon_\gamma$. For observers momentarily at rest, so that $u^\alpha = g_{00}^{-1/2}\delta_0^\alpha$ and $\varepsilon_\gamma = p^0 g_{00}^{1/2}$, integration of equation (6) yields

$$\int \hat{g}^{1/2} n_\gamma g_{00}^{-1/2}\, dx^2 \equiv N_\gamma = \text{const} \tag{7}$$

where $\hat{g} = |\det g_{\mu\nu}|^{1/2}$. The number of photons $N_\gamma$ is therefore conserved independently of the value of $g$.

Furthermore, we find that the cross section $A$ of the beam satisfies the relation $A_{;\alpha}\beta^{2-g}p^\alpha = A(\beta^{2-g}p^\alpha)_{;\alpha}$, which, using equations (4) and (6), yields $n_\gamma A/\nu = \text{const}$. As in standard theory, this expresses the conservation of the number of photons passing through different cross sections along the beam.

## 2.3. Adiabatic Photon Gas

Consider blackbody radiation in a box of volume $V$. Integrating equation (5) with $p = n_\gamma kT = 1/3\rho_\gamma$, one obtains

$$\rho_\gamma \sim \beta^{g-2} V^{-4/3}, \qquad N_\gamma T V^{1/3} \beta^{2-g} = \text{const} \tag{8}$$

The photon number $N_\gamma$ is defined as $n_\gamma V = \rho_\gamma V/\varepsilon_\gamma$. Since the frequency $\nu$ of the standing photon waves is related to $V$ by $\nu \sim V^{-1/3}$, it follows that

$$N_\gamma = \text{const} \tag{9}$$

independently of $g$. Therefore, the number of blackbody photons in a box is constant independently of whether the SEP is violated or not. We note that the relation $N_\gamma \sim \beta_a^{g-1}$ used in Canuto and Hsieh (1979) is consistent with equation (9) only if $g=1$. From equation (8) with the help of equation (9) it follows that

$$\rho_\gamma \sim \beta^{3(2-g)} N_\gamma^4 T^4 \sim \beta^{3(2-g)} T^4 \tag{10}$$

which reduces to the standard Stefan–Boltzmann relation for $g=2$.

### 2.4. Blackbody Radiation Spectrum

To obtain the differential spectral density $\rho_{\gamma\varepsilon}$ (with $\rho_{\gamma\varepsilon}\,d\varepsilon = \rho_{\gamma\nu}\,d\nu$ in an obvious notation), we use equation (8) in the form $\beta^{2-g} V^{4/3} \rho_\gamma = \text{const}$, whence (with $N_\gamma$ constant)

$$\begin{aligned} \rho_{\gamma\varepsilon} &\sim \beta^{3(2-g)} \varepsilon_\gamma^3 f(\varepsilon_\gamma/T) \\ \rho_{\gamma\nu} &\sim \beta^{g-2} \nu^3 f(\nu/\beta^{2-g} T) \end{aligned} \tag{11}$$

where $f(x)$ is a universal function of $x$. Putting $g=1$, we find that the second of equations (11) reduces to equation (5.17) of Canuto and Hsieh (1979). With $g=2$, the spectra are identical with standard spectra.

### 2.5. Photons and Massive Particles

Our conclusion [equation (9)] that the photon number is constant must be contrasted with our earlier result (Canuto and Goldman, 1982*a*) for the particle number $N_p$,

$$N_p \sim \frac{1}{\beta_a G} \sim \beta_a \qquad (g=2) \tag{12}$$

What this implies is that while all photon relations are unaffected by the presence of an SEP violation, massive particle numbers are affected. To take the comparison further, consider a system of massive particles described by $T_{\alpha\nu} = (p+\rho) u_\alpha u_\nu - p g_{\alpha\nu}$, where $\rho = \rho_0 + \rho_*$ and $\rho_0 = mn$, where $n = N_p/V$ and $p = \Gamma \rho_* = nkT$. Integrating equation (5), and using the expressions for $\rho_0$ in equation (4.23) of Canuto and Hsieh (1979) or equation (2.47) of Canuto *et al.* (1977), we conclude that

$$\rho_* V^\gamma \beta^\Omega = \text{const} \qquad \text{or} \qquad N_p T V^{\gamma-1} \beta^\Omega = \text{const} \tag{13}$$

where $\Omega = 3\Gamma + 1 - g$, $\gamma = 1 + \Gamma$. Eliminating $T$, one further gets (in AU)

$$p \sim \rho_0^\gamma N_p^{-\gamma} \beta_a^{-\Omega} \qquad \text{or} \qquad p \sim \rho_0^\gamma \beta_a^{-(\gamma-1)(2+g)} \tag{14}$$

showing that the $p$–$\rho_0$ relation is indeed altered by possible violation of the SEP. Equation (14) was used in Canuto (1981) to study the problem of the Earth's paleoradius.

The results of the preceding calculations may be summarized as follows:

(1) The photon number $N_\gamma$ and the particle number $N_p$ behave differently as a consequence of an SEP violation. While $N_\gamma$ is

constant as in equations (7) and (9), $N_p$ is not, equation (12), since $g$ must equal 2 if gravitational and atomic clocks (Canuto and Goldman, 1982*a*) are not to be equivalent.

(2) For $g=2$, all photon relations are unaffected by SEP violation.

(3) Accordingly, SEP violation cannot be detected by the study of free photons, because it affects only the behavior of massive particles.

## 3. COSMOLOGY

In our previous work (for example, Canuto *et al.*, 1979) we have compared the predictions of our earlier calculations with various cosmological data, especially those pertaining to the variation of magnitude or angular diameter with redshift for galaxies (including radiogalaxies) and for QSOs. [The results of Canuto *et al.*, (1979) and Canuto and Owens (1979) apply, provided that $g=2$.] These comparisons show that an SEP violation is not inconsistent with the data provided that $G \sim t^{-1}$ or, equivalently, that $\beta_a \sim t^{1/2}$. We now proceed to a consideration of the implications of our new results.

## 4. AGE OF STARS AND GLOBULAR CLUSTERS

If $G$ were larger in the past, stellar luminosities would have been great. For the sun, detailed numerical computations are available in the literature (Pochoda and Schwarzschild, 1964; Van den Berg, 1976, 1977; Maeder, 1977; Chin and Stothers, 1975; Carignan *et al.*, 1979) for the two cases $M \sim t^0$ and $M \sim t^2$, where $M$ is the total mass, both using $G \sim t^{-1}$. In the first case, the radioactive age of the sun can be matched if the hydrogen and metal abundances $X$ and $Z$, respectively, are $X = 0.82$ and $Z = 0.017$, whereas for $M \sim t^2$, the required abundances are $X = 0.725$–$0.73$ and $Z = 0.02$. The first set is considered unacceptable, since present estimates of the helium abundance $Y$ are $0.20 \le Y \le 0.30$, whereas the second set is clearly acceptable. In the present framework, because of $G\beta_a^2 = \text{const}$ and $\beta_a GM = \text{const}$ (Canuto *et al.*, 1977), we have $M(t) \sim G^{-1/2}(t) \sim t^{1/2}$, a case intermediate between those investigated numerically so far. The final result must therefore be $0.73 < X < 0.82$, leading to an acceptable helium abundance $Y$.

For globular clusters, unlike the case of the sun, the HR diagram provides a well defined turnoff position, and so since a larger luminosity in the past implies a shorter lifetime, one may avoid the difficulties created by an increase in the Hubble constant to 80–100 km sec$^{-1}$ Mpc$^{-1}$, as recently suggested. In fact, such large values would imply a maximum age of the universe of $(10$–$12) \times 10^9$ years, lower than the age of some globular clusters computed from standard theory, the estimated age of $15 \times 10^9$ years for M15, for example.

## 5. PAST TEMPERATURE OF EARTH

A larger $G$ in the past implies a higher solar luninosity $L$ and hence a higher effective temperature $T$ on the surface of the earth. Quantitative estimates of the effect, first done by Teller (1948), lead to

$$L \sim G^8, \qquad T \sim G^{2.5} \tag{15}$$

This implies that $4 \times 10^9$ years ago, $T = 500$ K, well above the boiling point of seawater and in apparent contradiction with well-substantiated data that liquid water existed on the earth as long as $3.8 \times 10^9$ years ago. We have used the power law $G/G_0 = t_0/t$, $t_0 = 15 \times 10^9$ years as well as the present temperature $T_0 = 250$ K, corresponding to no greenhouse effect. Also included in the estimate of $T = 500$ K is the prediction by standard stellar evolution (Newman and Rood, 1977) that $4 \times 10^9$ years ago the sun's luminosity was 30–40% lower than today, implying an 8% reduction in $T$.

Within the present framework, the equations for the hydrostatic equilibrium equation, the radiative transfer, and stellar luminosity [$L = \varepsilon M$, where $\varepsilon \sim \rho T^n$ is the nuclear energy rate (Pochoda and Schwarzschild, 1964; Van den Berg, 1976, 1977; Maeder, 1977; Chin and Stothers, 1975; Carignan *et al.*, 1979)] and the equations of state $pV = N_p kT$, $\rho_\gamma \sim T^4$, together with $G = G(t)$ and $M \sim G^{-1/2} \sim \beta_a$, yield by homology arguments

$$L \sim G^\alpha M^\delta \sim G^{s/2} \sim \beta_a^{-s}$$

where

$$s = 2\alpha - \delta = \frac{59 + 18n}{5 + 2n} \tag{16}$$

With $n = 4$ for the sun, and remembering that the earth–sun distance $D$ scales like $D_0 = \beta_a D$, we derive

$$L \sim G^5, \qquad T \sim G \tag{17}$$

which implies that $4 \times 10^9$ years ago, $T \approx 313$ K, well below the boiling point of seawater.

Although a fully reliable result must await an atmospheric calculation as well as the evaluation of the $L$–$G$ relation based on a detailed stellar evolutionary model, the estimates presented here indicate the contrast with the results obtained from previous quantifications of the SEP violation.

Predictions of "boiling oceans," which contradict existing data, do not arise in the present model. [For the predictions of the Brans-Dicke theory see Dicke (1964).]

## 6. 3 K BLACKBODY RADIATION

The blackbody radiation has long been considered of crucial importance to SEP-violating theories. One important consideration must be stressed—*a priori*, it is not known whether an SEP violation requires the standard particle conservation law be changed and, if so, whether photons and massive particles are equally affected.

The first theory implementing an SEP violation, that of Jordan (1955), made no predictions on this problem, thus leaving the question to be settled by observational data. Honl and Dehnen (1968) showed that a nonconstant photon number $N_\gamma$ would be incompatible with the 3 K blackbody spectrum, whereupon Jordan (1968), acknowledging the impossibility of reconciling his theory with variable $N_\gamma$, suggested that the other version of his theory with constant $N_\gamma$ be adopted. (This coincides with the Brans-Dicke theory.)

Perhaps unaware of the 1968 results, Dirac (1972, 1973, 1975) stated that a variable $N_\gamma$ cannot be reconciled with the 3 K radiation. The same conclusion was reached by Canuto and Lodenquai (1977) in an attempt to test an SEP violation by the use of observational data. Again unaware of the 1968 work, Steigman (1978) arrived at the same conclusion as Honl and Dehnen, namely that a nonconstant $N_\gamma$ is not compatible with the 3 K radiation.

Thereafter Canuto and Hsieh proposed two alternative solutions of the 3 K problem. In 1978, it was shown (Canuto and Hsieh, 1978) that if instead of the gauges $G \sim \beta_a^{\pm 1}$ $(G \sim t^{-1})$ previously used until then by Dirac, Jordan, and so on, one chose (1) $G \sim \beta_a^{-2}$ together with (2) $\varepsilon_\gamma \sim \nu$ and (3) $T \sim R^{-1}$, the resulting $N_\gamma$ was constant and that the 3 K radiation does not exclude a violation of the SEP. However, since the necessary conditions (1)-(3) were assumed, not derived from a complete photon theory, it could not be claimed that the 3 K problem had been solved. In 1979, a complete photon theory was presented (Canuto and Hsieh, 1979) in which the propagation equation, the single-particle relations, and the thermodynamic relations were all derived in a consistent manner. The result was that the 3 K radiation is compatible with an SEP violation provided (1) $N_\gamma \sim \beta_a^{g-1}$, (2) $\varepsilon_\gamma \sim \nu/\beta_a$, and (3) $T \sim \beta_a^{-1} R^{-1}$.

In 1982, however, it was first realized (Canuto and Goldman, 1982*a*) that a violation of the SEP demands $g = 2$. Since the photon theory of (Canuto and Hsieh, 1979) has been shown here to be valid for $g = 1$ (in spite of the original belief that it held for any $g$), the theory is no longer tenable. Unlike Jordan's theory, the 1979 photon theory has to be abandoned, not because of observational reasons, but because of theoretical consistency requirements.

In the theory presented here, we have shown that $N_\gamma$ remains constant for any $g$ and that for the required value $g=2$, all photon relations are identical with the standard ones. Photons are therefore unaffected by an SEP violation, and the 3 K radiation imposes no constraints on our formulation of the SEP violation.

Finally, we note that both Jordan's theory with variable $N$ (both $N_\gamma$ and $N_p$) and that of Brans–Dicke with both $N_p$ and $N_\gamma$ constant are not viable, although for different reasons. Our theory, with $N_\gamma = \text{const}$, but $N_p \neq \text{const}$, has thus far not encountered such difficulties.

## 7. NUCLEOSYNTHESIS

The nucleosynthesis of light elements has been a most valuable tool to probe the early universe (Weinberg, 1972), and attempts have been made to use the $^4$He abundance to set limits on the violation of the SEP, expressed phenomenologically as a variable $G$.

The most recent study (Rothman and Matzner, 1982) indicates that if the shape of $\beta_a$ as a function of $t$ determined from the matter era is extrapolated to the radiation era, unacceptable values of $^4$He are obtained. Indeed, the nucleosynthesis data demand an almost constant $\beta_a$ during the radiation-dominated era, and for good reasons. The function $\beta_a$ represents a cosmological influence on local physics (Canuto and Goldman, 1982*a*,*b*). The dynamics of $\beta_a$ is therefore determined by the global structure of the universe, which in turn is governed by the energy density (and pressure) of matter and radiation. Since matter is affected by $\beta_a$, equations (12)-(14), while radiation is not, it is plausible that during the matter-dominated era, the rate of variation in $\beta_a$ is comparable with the rate of expansion—that is, $\dot{\beta}_a/\beta_a \sim 1/t$, or $\beta_a \sim t^{-n}$, with $n \sim 1$. However, during the radiation era, such a variation is expected to have been considerably smaller, that is, $\dot{\beta}_a/\beta_a \ll 1/t$, since the dynamical part played by matter was negligible. In such a case, the value of $\beta_a$ during the short radiation period would not have differed significantly from, say, the last "matter-dominated" value at decoupling. Indeed, the main result of Rothman and Matzner (1982) is that during the radiation era $\dot{\beta}_a/\beta_a \ll 1/t$, that is, $\beta_a \sim t^{-n}$, $n \ll 1$.

We shall therefore propose that during the radiation era, $\beta_a$ was essentially constant and investigate the consequences for nucleosynthesis. Using the standard framework, Olive *et al.* (1981) have recently analyzed the dependence of the calculated abundances of the most relevant parameters, namely (1) the baryon-to-photon number ratio $\eta = n_B/n_\gamma$, which in standard cosmology is equal to its present value $\eta_0 = \rho_{B0} \cdot 10^{22} (2.7/T_0)^3/6.64$, where $\rho_{B0}$ is expressed in g cm$^{-3}$, (2) the neutron half-life $\tau_{1/2}$, and (3) a speedup factor $\xi$, defined by $\dot{R}/R = \xi(\dot{R}/R)_{\text{standard}}$.

In the SEP-violating framework, a constant $\beta_a$ affects nucleosynthesis in two ways. Because of $N_p \sim \beta_a$, we have $n_B \sim \beta_a R^{-3}$, while $n_\gamma \sim R^{-3}$. Therefore $\eta = \eta_0\beta_a$. Furthermore, the RW equation (Canuto *et al.*, 1979) with $G\beta_a^2 = \text{const}$ and $k = 0$, becomes

$$\frac{\dot{R}}{R} = \beta_a^{-1}\left(\frac{8\pi G_0}{3}\rho_\gamma\right)^{1/2} = \beta_a^{-1}\left(\frac{\dot{R}}{R}\right)_{\text{standard}}$$

so that $\beta_a^{-1}$ plays the part of a speedup factor $\xi$.

In Figure 1 the calculated $^4$He ($Y$) and D ($D$) abundances are plotted as a function of $\eta_0$ for the standard ($\beta_a = 1$) and the present theory. In both cases, $N_\nu = 3$ and $\tau_{1/2} = 10.6$ min. The standard model $^4$He and D abundances were taken from Olive *et al.* (1981) and Yang *et al.* (1979), respectively. The $^4$He and D abundances corresponding to the present model were calculated from Figure 2a of Olive *et al.* (1981) using $\xi \equiv \beta_a^{-1}$, and $\eta = \eta_0\beta_a$ and from Table 1 of Yang *et al.* (1979) [the D abundance is a function of $\eta/\xi = \eta_0\beta_a^2$ (Yang *et al.*, 1979)]. We first consider the results of the standard case. Determinations of the $^4$He abundance $Y$ in recent years have limited the uncertainty to the range $0.25 \gtrsim Y \gtrsim 0.20$, corresponding to $4.5 \times 10^{-11} \lesssim \eta_0 \lesssim 4 \times 10^{-10}$. This in turn implies that the D abundance must be $8 \times 10^{-5} \lesssim \text{D} \lesssim 2.5 \times 10^{-3}$. The presently observed D abundance is estimated (Steigman, 1982) to be in the range $(1\text{-}4) \times 10^{-5}$, an interval which may still be consistent with that given above since it must be a lower limit because D is in fact only destroyed by stellar processes. The quantification of the destruction is difficult. On the one hand, Rana (1982) has concluded that the primordial D abundance must have been $\sim < 6 \times 10^{-5}$, implying that the standard framework is inconsistent. On the other hand, Yang *et al* (in preparation) and Steigman (1982) have argued that since D is converted into $^3$He, the combined D+$^3$He should not be less than its present value. This leads to $\eta_0 \gtrsim 2 \times 10^{-10}$, which in turn corresponds to a D abundance $\lesssim 2.5 \times 10^{-4}$, in agreement with the range previously determined. Even if this second solution is accepted, there may be a further problem. Recent data (Olive *et al.*, 1981; Rayo *et al.*, 1982) on the $^4$He abundance have suggested values as low as 0.22, implying a D abundance $> 1.4 \times 10^{-3}$, a value which may be difficult to accommodate (Stecker, 1980; Olive and Turner, 1981). (Note that the lower the $^4$He abundance, the higher the D abundance.)

Finally, another potential source of inconsistency stems from the constraint (Olive *et al.*, 1981) $\eta_0 \gtrsim 2 \times 10^{-10}$, reached by considerations of dynamics of binary galaxies and small groups of galaxies. This value implies $Y \gtrsim 0.24$, in contradiction with $Y = 0.22$, should this last value be confirmed

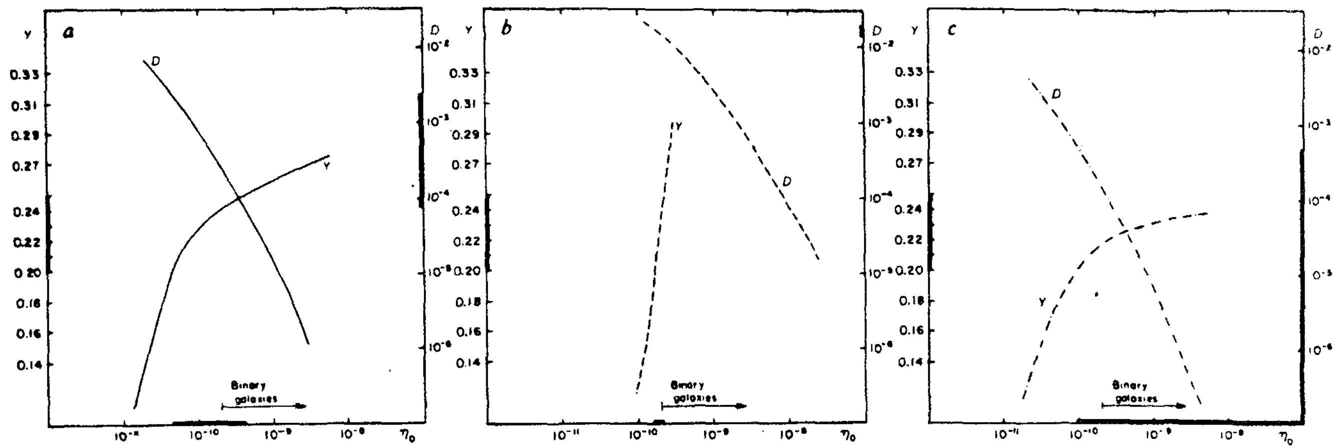


**Fig. 1.** Predicted helium ($Y$) and deuterium ($D$) abundances by mass. Given the range of observed values of $Y$, one determines the corresponding baryon to photon number ratio $\eta_0$. This in turn is projected under the $D$ curve and a $D$ abundance is arrived at, which is then compared with observations. If one accepts the most recent D abundances $(7.0\text{–}3.4)\times 10^{-5}$ (Gautier and Owen, 1983), it is seen that the $\beta_a = 1.2$ case fits the data better than the standard case $\beta_a = 1$. (a) $\beta_a = 1$; (b) $\beta_a = 0.2$; (c) $\beta_a = 1.2$.

by future observations. [Here, too, a solution has been proposed (Olive *et al.*, 1981; Steigman, 1982): massive neutrinos, which by contributing to the masses of galaxies would allow a lower value of $\eta_0$ due to baryons and so a lower $Y$.]

In conclusion, while it is premature to claim that inconsistencies have been found, it may turn out to be difficult to accommodate $Y$, $D$, and the constraints of galaxy dynamics within the standard SEP-conserving framework.

In the present framework, and on the basis of available lunar data suggesting $\dot{\beta}_a > 0$, we assume that $\beta_a$ is monotonically increasing during the matter-dominated era. We have suggested a constant $\beta_a$ in the radiation era. If $\beta_a$ behaves monotonically during the radiation-to-matter transition period, then the constant value must be less than its present value and presumably not very different from its value at the transition time. It is, however, possible that the change of scenario during the transition period and the different behavior of matter and radiation as a function of $\beta_a$ may have caused a nonmonotonic behavior during the transition period, thus allowing for a constant greater-than-unity value throughout the radiation-dominated epoch. The implications for nucleosynthesis of these two alternatives were examined by considering two representative values of $\beta_a$, namely $\beta_a = 0.2$ and $\beta_a = 1.2$ (see Figure 1).

$\boldsymbol{\beta_a = 0.2}$. Repeating the same arguments as in the standard case, we see that the inconsistency between $\eta_0 \geq 2 \times 10^{-10}$ (binary galaxies) and $Y = 0.23$ no longer exists. The predicted value of $D$ would be about $10^{-2}$, a factor of 10 larger than the value predicted by the standard framework if indeed $Y = 0.23$.

$\boldsymbol{\beta_a = 1.2}$. In this case, all the demands for low $Y$, $D$, and $\eta_0$ (binary galaxies) can be satisfied and no inconsistencies arise. For $Y = 0.22$–$0.23$, $\eta_0$ is required to lie between $(2\text{–}6.5) \times 10^{-10}$, in agreement with the limit contained from binary galaxies. At the same time, $D$ would be $(0.1\text{–}1.5) \times 10^{-5}$, again in agreement with the present data.

## 8. COMPARISON WITH PREVIOUS SCHEMES

Two features that differentiate our framework from previous SEP-violating schemes are important.

First, they generally assume that (in AU) a violation of the SEP will affect only gravitational physics while leaving all nongravitational relations unaffected. In the present theory, gravitational physics is affected by $\beta_a$, but so are the nongravitational many-body relations for massive particles; for massless particles, all relations are independent of $\beta_a$. [This results in a different $G$ dependence of several of the tests, leading in general to a weaker dependence on $G$; see equation (17).]

The second and perhaps more important difference is that the present theory allows $\dot{\beta}_a/\beta_a \sim \Lambda H_0$, with $\Lambda \sim 1$, which is a violation of the SEP of the order of the Hubble constant, as expected on physical grounds for a violation of cosmological origin. By contrast, $\Lambda \sim 1$ cannot be achieved in other SEP-violating theories, where it turns out that $|\Lambda| \lesssim 10^{-3}$. [In the Brans-Dicke theory (Weinberg, 1972), $\Lambda \sim (\omega+2)^{-1}$ and (Reasenberg *et al.*, 1979) $\omega > 500$; in the Rosen theory (Goldman and Rosen, 1977), $\Lambda \sim \alpha_2$ and (Warburton and Goodkind, 1976) $|\alpha_2| \lesssim 10^{-3}$.]

## 9. CONCLUSIONS

We have attempted to test a possible SEP violation against already existing data, in the hope of providing an incentive for a direct experimental test. To that end, we have constructed a theoretical framework in which the SEP violation is represented by a function $\beta_a$ treated phenomenologically that is not provided by the theory itself, much as viscosity or heat conduction coefficients are treated in classical fluid dynamics. The implications of $\beta_a$ on gravitational and nongravitational phenomena are then checked against observational data ranging from astrophysics to geophysics and limitations on the variability of $\beta_a$ arrived at.

Specifically, we have constructed a scale-covariant (unit-independent) formalism (Canuto *et al.*, 1977), a mathematical procedure that *per se* does not introduce new physics, in much the same way that coordinate covariance requirement does not (Kretschmann, 1917; Einstein, 1918). In our case the physical input occurs when we define the properties that characterize gravitational as different from atomic units. [See Canuto and Goldman (1982*a*) for the definitions of the two units.]

In our previous paper (Canuto and Goldman, 1982*a*) we showed that an SEP violation occurs when atomic and gravitational clocks run at different rates, which occurs provided that (1) the relation between the function $\beta_a$ and the gravitational coupling $G$ is $G\beta_a^2 = \text{const}$ and that (2) the equations of motion of microscopic particles are different from those of macroscopic bodies. This last property implies that one- and many-body systems respond differently to an SEP violation. In fact, while one-particle relations are unchanged, many-body relations for massive particles depend on $\beta_a$ [equations (12) and (14)].

Our present treatment of massless particles under an SEP violation shows that the photon number $N_\gamma$ is conserved and that because of the relation $G\beta_a^2 = \text{const}$, all photon relations (single- and many-body) are unaffected by $\beta_a$—they coincide with the standard expressions. The physical implication of this result is that a system of free photons cannot reveal the possible existence of an SEP violation, a result important for the study of the 3 K blackbody radiation and nucleosynthesis in the radiation-dominated era.

Using the earlier results for massive particles (Canuto and Goldman, 1982*a*) together with those obtained here, our compatibility tests ranging from nucleosynthesis to the radius of the earth 400 Myr ago (Canuto, 1981) show that all the data we have analyzed are consistent with an SEP violation of the order of the Hubble constant during the matter-dominated era, $\dot{\beta}_a/\beta_a \sim 1/t$, and with $\dot{\beta}_a/\beta_a \ll 1/t$ during the radiation-dominated era.

Thus, our conclusion that an SEP violation, while not demanded, is compatible, will hopefully stimulate a direct experimental search using the best data available—the Viking data (Canuto and Goldman, 1982*b*; Canuto *et al.*, in press; Adams *et al.*, submitted).

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